$10^2 \times 10^3 = 10^{2+3} = 10^5$

b. **Example.** Multiply 3.2×10^2 by 2.0×10^3

Solution. Remember that when multiplying exponential expressions, the digit terms are multiplied in the usual manner and the exponents of the exponential terms are added, therefore:

$(3.2 \times 10^2) \times (2.0 \times 10^3) = 3.2 \times 2.0 \times 10^2 \times 10^3$

$= 3.2 \times 2.0 \times 10^{2+3} = 6.4 \times 10^5$

c. **Example.** Multiply 3.2×10^{-2} by 2.0×10^3

Solution.

$(3.2 \times 10^{-2}) \times (2.0 \times 10^3) = 3.2 \times 2.0 \times 10^{-2} \times 10^3$

$= 6.4 \times 10^{-2+3} = 6.4 \times 10^1$

Equivalently:

$$\begin{array}{l} 3.2 \times 10^{-2} \\ \underline{2.0 \times 10^{3}} \\ 6.4 \times 10^{1} \end{array}$$

Multiply the digit terms and add the exponents of the exponential terms.

d. **Example.** Multiply 0.00056 by 0.013 using scientific notation.

Solution. Before we can multiply, the above numbers must be converted to scientific notation.

Thus, $0.00056 = 5.6 \times 10^{-4}$ and $0.013 = 1.3 \times 10^{-2}$

$$\begin{array}{l} 5.6 \times 10^{-4} \\ \underline{1.3 \times 10^{-2}} \\ 7.28 \times 10^{-6} \end{array} \text{ ----> } 7.3 \times 10^{-6}$$

1-21. DIVISION OF NUMBERS EXPRESSED IN SCIENTIFIC NOTATION

a. **Method One.** To divide exponential expressions, the digit term of the numerator is divided by the digit term of the denominator in the usual way, and the exponent of the denominator is subtracted from the exponent of the numerator.

(1) Example. Divide 10^3 by 10^2

Solution. In division, the exponent of the denominator is subtracted from the exponent of the numerator. Therefore:

10^3 divided by $10^2 = 10^{3-2} = 10^1 = 10$

(2) Example. Divide 5.0×10^3 by 2.5×10^2

Solution. By using method one, the laboratory specialist divides 5.0 by 2.5 and then subtracts the exponents of the exponential terms. Therefore:

$$\frac{5.0}{2.5} = 2.0 \text{ and } 10^{3-2} = 10^1$$

Thus, 5.0×10^3 divided by $2.5 \times 10^2 = 2.0 \times 10^1$

b. **Method Two.** In this method of dividing exponential expressions, the digit term is treated in exactly the same manner as in method one. The exponential term in the denominator is moved to the numerator, with the appropriate sign change, and the exponents of the exponential terms in the numerator are added.

(1) Example. Divide 5.0×10^3 by 2.5×10^2

Solution.

$$\frac{5.0 \times 10^3}{2.5 \times 10^2} = \frac{5.0 \times 10^3 \times 10^{-2}}{2.5} = \frac{5.0 \times 10^{3-2}}{2.5} = 2.0 \times 10^1$$

(2) Example. Divide 0.000005 by 250

Solution. Before the above numbers can be divided using the methods previously demonstrated, they must be converted to scientific notation.

Thus, $0.0000050 = 5.0 \times 10^{-6}$ and $250 = 2.5 \times 10^2$

$$\frac{5.0 \times 10^{-6}}{2.5 \times 10^2} = \frac{5.0 \times 10^{-6} \times 10^{-2}}{2.5} = 2.0 \times 10^{-8}$$

1-22. EXERCISES, SECTION III

After you have completed these exercises, turn to the end of the lesson, and check your answers with the review solutions.

FIRST REQUIREMENT: Express the following numbers in standard scientific notation:

a. 0.002406

____

b. 4742

____

c. 1.463

____

d. 774.82

____

e. 91

f. 0.2534

SECOND REQUIREMENT: Evaluate the following expressions:

g. $10^2 + 19$

j. $(10^1)^3$

h. $2^3 2^4$

k. $\frac{3^3}{3^0}$

i. $2^2 3^2$

l. $\frac{8^2}{8^{-2}}$

THIRD REQUIREMENT: Evaluate the following expressions:

m. 4.0×10^{-11} divided by 2.0×10^{-3}

n. 8.4×10^6 divided by 2.1×10^{-3}

o. 3.4×10^5 multiplied by 2.2×10^2

p. 6.119×10^1 multiplied by 1.112×10^{-1}

FOURTH REQUIREMENT: Write expressions that change the signs of the following exponentials without changing the value:

q. $\frac{1}{7^{-4}}$

r. 10^{-16}

s. $\dfrac{5.5 \times 10^{-3}}{9.1 \times 10^{6}}$ ______________________

t. $\dfrac{10^{-8}}{2^{4}}$ ______________________

Section IV. LOGARITHMS

1-23. DISCUSSION

The "common" logarithm (log) or the logarithm to the base 10 ($\log_{10}$) of a number is the exponent (power) to which the number ten (10) must be raised to equal that number. For example, the logarithm of 100 is equal to two (2), since the exponent (power) to which the number ten (10) must be raised to equal 100 is two (2) or $100 = 10^2$. Since logarithms are exponents, they follow the "Rules of Exponentiation" previously discussed. The laboratory specialist can utilize logarithms to perform multiplication, division, find roots, and raise a number to a power. A second use of logarithms is in the solving of a number of equations used in the clinical laboratory; e.g., pH = $-\log [H^+]$ and absorbance = 2 - log %T. Logarithms exist only for positive numbers.

NOTE: The work with logarithms in this review will consist of traditional manual methods with tables rather than the use of a calculator to find logarithms and related values. Please remember this when you do the exercises and the examination items. Logarithms are approximate values. Comparable operations using a calculator will yield slightly different results in most instances.

1-24. EXAMPLES

Number	Number expressed exponentially	Logarithm (log)
10000	10^4	4
1000	10^3	3
100	10^2	2
10	10^1	1
1	10^0	0
0.1	10^{-1}	-1
0.01	10^{-2}	- 2
0.001	10^{-3}	- 3
0.0001	10^{-4}	-4

It is easily seen that the exponent (power) to which the number ten (10) has been raised is the same as the logarithm.

1-25. PARTS OF A LOGARITHM

A logarithm is composed of two parts:

a. The characteristic is the portion of the logarithm that lies to the left of the decimal point. It is a whole number that may be negative or positive, depending upon the original number.

b. The mantissa is the portion of the logarithm that lies to the right of the decimal point. The mantissa is a decimal fraction of one (1) and it is always positive.

c. For example, the logarithm of 20 is equal to 1.3010, where the number one (1) is the characteristic and the number .3010 is the mantissa.

1-26. DETERMINATION OF THE CHARACTERISTIC

In order to find the logarithm of a number, the characteristic must first be determined. The characteristic is always determined by inspection of the original number or by applying a simple rule.

a. **By Inspection.**

(1) If we take the number 20, we see that it lies between 10 and 100, which have logarithms of one (1) and two (2), respectively. Therefore, the logarithm of the number 20 must be between one (1) and two (2). The logarithm of 20 is 1.3010 and hence the characteristic (remembering that the characteristic is the number to the left of the decimal point) of the logarithm is one (1).

(2) An alternate method is to express the original number in scientific notation. The power of ten (10) in the expression is the characteristic of the original number. The number 20 when expressed in scientific notation is 2.0×10^1. Hence, the characteristic of the original number is one (1).

NOTE: Both methods for determining the characteristic may be used for numbers less than or greater than one.

b. **By Rule.**

(1) For original numbers greater than one (1): If the number whose logarithm is being determined is greater than one (1), the characteristic for the logarithm of that number will be positive and one less than the number of digits found to the left of the decimal point in the original number. For example:

Original number	Number of digits to left of decimal	Characteristic (digits - one)
2.0	1	0
20.0	2	1
200.0	3	2
2000.0	4	3
20000.0	5	4

(2) For original numbers that are less than one (1): If the number whose logarithm is being determined is less than one (1), the characteristic for the logarithm of that number will be negative and is a number one more than the number of zeros (0's) to the right of the decimal point that precede the first nonzero integer. For example:

Original number	Number of digits to right of decimal	Characteristic (num. of 0 + 1)
0.2	0	-1
0.02	1	-2
0.002	2	-3
0.0002	3	-4

(3) A negative characteristic may be distinguished from a positive characteristic by placing a negative sign above the characteristic. This is necessary since the mantissa is always positive. For example:

$$\log 0.2 = \bar{1}.3010$$

(4) Equivalently, the negative logarithm of 0.2 could be expressed as the difference between the characteristic and mantissa. This is done to facilitate certain methods of calculation, pH in particular. In this case, the negative sign is placed in front of the difference. For example:

$$\log 0.2 = \bar{1}.3010 = 0.3010 - 1 = -0.6990$$

This method is considered by most authors to be an incorrect method to express the logarithm of a number less than one, chiefly due to the fact that the mantissa may never be negative. However, this method is commonly used by electronic calculators and certain other methods of calculation. You should be able to use either method without error in the laboratory as long as you consistently use either approach.

(5) A logarithm with a negative characteristic may also be changed to a positive form by adding ten (10) to the characteristic and adding minus (-) ten (10) after the mantissa. Since ten (10) is both added and subtracted to the logarithm the operation does not change the value of the logarithm. For example:

$$\log 0.2 = \bar{1}.3010$$

Add ten (10) to the characteristic

$$10 + (-1) = 9 \dashrightarrow 9.3010$$

Add minus ten (-10) after the mantissa

$$9.3010 - 10 = \bar{1}.3010$$

1-27. DETERMINATION OF THE MANTISSA

The mantissa of the logarithm is found in the table of "common" logarithms (see Appendix B) and is independent of the position of the decimal point in the original number. Thus, the numbers 0.02, 0.2, 2, 20, 200, 2,000, and 20,000 all have the same mantissa; i.e., 0.3010.

a. In determining the number to obtain the mantissa, do not consider preceding zeros (0).

b. The number for which a mantissa is desired, commonly referred to as the natural number, must contain at least three digits, the last two of which may be zeros (0).

c. For numbers that contain less than three digits, add enough zeros (0) to yield a three digit number.

d. Numbers that contain more than three nonzero digits should be rounded to three significant figures.

e. Consider the following examples:

Original Number	Base Number
0.312	312
1.031	103
55.357	554
4	400
0.9	900

f. Examine the Appendix B, Table of Four-Place Logarithms. The table consists of a series of columns and rows of four-digit numbers bordered left by a column of two-digit numbers and uppermost by a row of single-digit numbers.

g. The first two digits of the natural number can be found in the first column.

h. The third digit of the number can be found in the uppermost row.

i. To determine the mantissa of a number, find the first two digits of the number in the left column. Move across this row to the column headed by the third digit of the number.

j. The four-digit number at the intersection is the mantissa of the given natural number.

1-28. DETERMINATION OF THE LOGARITHM OF A NUMBER

Calculators are the most common means of determining the logarithm of a number. The keystroke sequence varies dependent upon the type and model of the calculator. It will be the student's responsibility to become familiar with his or her calculator. In this review section use both the tables and a calculator, if available, to determine the logarithm of a number.

Instructions on the use of a calculator will be provided by the staff when you arrive for the 6F-66F course.

To determine the logarithm of a number both the characteristic and the mantissa must be found. The characteristic must be determined by inspection (or rule), and the mantissa must be looked up in a table of "common" logarithms (a calculator may also be used). By close examination of the following table, the determination of the logarithm of a number should become clear.

Original number	Characteristic	Mantissa	Logarithm
0.2	-1	3010	$\bar{1}.3010$
20	1	3010	1.3010
200	2	3010	2.3010
2000	3	3010	3.3010
20000	4	3010	4.3010
0.030	-2	4771	$\bar{2}.4771$
412	2	6149	2.6149
5490	3	7396	3.7396

1-29. LOGARITHMS OF NUMBERS EXPRESSED AS EXPONENTIALS

The determination of the logarithm becomes quite simple when the original number is written in scientific notation; i.e., as an exponential expression. This form of expressing the original number also should help you understand the meaning of the characteristic and the mantissa.

a. **Example.** Find the logarithm of the number 4120.

Solution. First write 4120 in exponential form.

$$4120 \text{ becomes } 4.12 \times 10^3$$

Take the logarithm of the exponential expression.

$$\log (4.12 \times 10^3)$$

When determining the logarithm of two numbers being multiplied together (4.12×10^3), the logarithm of <u>each number</u> is determined and the logarithms are then added.

$$\log (4.12 \times 10^3) = \log 4.12 + \text{the } \log 10^3$$

The logarithm of 4.12 is 0.6149 (mantissa) and the logarithm of 10^3 is three (3) (characteristic). Thus, the logarithm of 4120 = 0.6149 + 3 or 3.6149.

b. **Example.** Find the logarithm of 0.000412.

Solution.

$0.000412 = 4.12 \times 10^{-4}$

$\log (4.12 \times 10^{-4}) =$

$\log 4.12 + \log 10^{-4} =$

$0.6149 + (-4) =$

$\bar{4}.6149$ or equivalently, with a calculator, -3.3851

1-30. DETERMINATION OF ANTILOGARITHMS

The antilogarithm (antilog) is the number corresponding to a logarithm. For example, the antilogarithm of 3 is the number whose logarithm equals three (3). The number whose logarithm equals three (3) is 1000; thus, the antilogarithm of 3 is 1000.

a. **Mantissa.** The mantissa of the logarithm can be found in the table of "common" logarithms, and the three digits corresponding to the mantissa is written down. **In the event that the exact mantissa value is not in the table, find the closest mantissa in the columns without exceeding the value of the mantissa being worked with.** Remember the mantissa is the number to the right of the decimal point.

Examples.

Mantissa	Corresponding Digits
0.8451	700
0.3010	200
0.2945	197
0.6981	499
0.9996	999

b. **Characteristic.** The characteristic of the logarithm determines where the decimal point will be placed in the number corresponding to the mantissa when determining the antilogarithm.

Examples.

Characteristic	Number of digits to the left of the decimal
0	1
1	2
2	3
3	4

c. **Example.** Find the antilogarithm of 2.000.

Solution. Determine the digits that correspond to the mantissa. The mantissa is .000 and the digits corresponding to this mantissa in the table of "common" logarithms are 100. The characteristic of two (2) indicates that there are three (3) digits or places to the left of the decimal point. Thus, antilog 2.000 = 100. To check your answer, take the logarithm of 100:

log 100 = 2

d. **Example.** Find the antilogarithm of 3.2989.

Solution. The digits corresponding to the mantissa (0.2989) are 199. The characteristic (3) indicates that there are four (4) digits to the left of the decimal point. Therefore, antilog 3.2989 = 1990.

e. **Example.** Find the antilogarithm of $\bar{3}.2989$.

Solution. Rewrite $\bar{3}.2989$ as 0.2989 - 3. This puts the logarithm in the form of an exponential expression. Take the antilogarithm of both numbers.

Antilog 0.2989 X antilog -3.

NOTE: The two antilog expressions are multiplied together since they represent numbers, not logs (refer to the Rules of Exponentiation in paragraph 1-17).

The antilogarithm of 0.2989 = 1.99

The antilogarithm of minus three (-3) is 10^{-3}

Thus, the antilogarithm of $\bar{3}.2989 = 1.99 \times 10^{-3}$

f. **Example.** Use the logarithm table to find the antilogarithm of -2.5017, a logarithm from an electronic calculator.

Solution. In the calculator, the negative characteristic and positive mantissa were combined by algebraic addition to form the single negative number. To change a negative logarithm to the correct form, add the next highest positive whole number above the characteristic's (absolute value) to the negative log. This operation will yield the positive mantissa.

$3 - 2.5017 = 0.4983$

Use the additive inverse (negative form in this instance) of the whole number that was added as the characteristic of the corrected logarithm, and indicate its sign with a bar over the number.

$\bar{3}.4983$

Rewrite $\bar{3}.4983$ as $0.4983 - 3$. This puts the logarithm in the form of an exponential expression. Take the antilog of both numbers.

Antilog 0.4983 X antilog (-3)

<u>NOTE</u>: The two antilog expressions are multiplied together since they represent numbers, not logs (refer to the Rules of Exponentiation in paragraph 1-20).

The antilog of 0.4983 = 3.15

The antilog of -3 = 10^{-3}

Therefore, the antilog of -2.5017 = 3.15 X 10^{-3}

1-31. USE OF LOGARITHMS IN MULTIPLICATION

To multiply two or more numbers together, the logarithms of each number is found, and the logarithms of each are added together. The antilogarithm of the sum of the logarithms is then taken to give the product of the multiplication.

Example. Multiply 152 by 63 using logarithms.

Solution.

log 152	=	2.1818
log 63	=	+ 1.7993
Sum		3.9811

antilog 3.9811 = 9.57 X 10^3

Thus, 152 X 63 ≐ 9.6 X 10^3, with 2 significant figures

<u>NOTE</u>: If you were to multiply these two numbers together by the usual method you would have obtained 9576 instead of 9570 as was determined by the use of logarithms due to the accuracy of the logarithm tables. <u>Logarithms are approximate values</u>.

1-32. USE OF LOGARITHMS IN DIVISION

To divide two numbers the logarithms of the numbers are subtracted; i.e., the logarithm of the divisor (denominator) is subtracted from the logarithm of the dividend (numerator). The antilogarithm of the difference of logarithms is then taken to give the quotient.

Example. Divide 152 by 63 using logarithms.

Solution.

log 152 =	2.1818
log 63 =	-1.7993
Difference	0.3825

antilogarithm of 0.3825 = 2.41

Thus, 152 divided by 63 = 2.4, with 2 significant figures

1-33. USE OF LOGARITHMS TO FIND ROOTS OF NUMBERS

To find the root of a number the logarithm of the number is determined; the logarithm of the number is next divided by the root desired; e.g., if the square root of a number is wanted, the logarithm of the number is divided by two (2); if the cube root is required, divide by three (3), etc. The antilogarithm of the quotient is taken; the resulting number is the root of the number.

a. **Example.** Find the square root of 625.

Solution.

Square root 625 = $(625)^{1/2}$

log $(625)^{1/2}$ = 1/2 log 625 = 1/2 X 2.7959 = 1.3980

antilogarithm 1.3980 = 25.0

Thus, the square root of 625 = 25.0

b. **Example.** Find the fourth root of 625.

Solution.

Fourth root 625 = $(625)^{1/4}$

log $(625)^{1/4}$ = 1/4 log 625 = 1/4 X 2.7959 = 0.6990

antilog 0.6990 = 5.00

Thus, the fourth root of 625 = 5.00

1-34. USE OF LOGARITHMS TO FIND THE PRODUCT OF NUMBERS WITH EXPONENTS

To find the product of a number that has an exponent, determine the logarithm of the number and multiply by the exponent. The antilogarithm of the product is taken. For example, if a number has been squared, find the logarithm of the number and multiply by two (2), then find the antilog.

a. **Example.** Find the product of $(625)^2$ using logarithms.

Solution.

$\log 625^2 = 2 \log 625 = 2 \times 2.7959 = 5.5918$

$\text{antilog } 5.5918 = 391{,}000 = 3.91 \times 10^5$

<u>NOTE</u>: If 625 is squared with a calculator, the resulting number is 390,625.

b. **Example.** Find the product of 625^{10} using logarithms.

Solution.

$\log 625^{10} = 10 \log 625 = 10 \times 2.7959 = 27.959$

$\text{antilog } 27.959 = 9.10 \times 10^{27}$

1-35. EXERCISES, SECTION IV

After you have completed these exercises, turn to the end of the lesson, and check your answers with the review solutions.

FIRST REQUIREMENT: Determine the logs for the following number.

a. 695,100

____

b. 821

____

c. 0.00063

____

d. 3421

____

e. 0.0458

____

SECOND REQUIREMENT: Determine the antilogs of the following logarithms.

f. 8.6990 - 10

____

i. 1.5400

____

g. -2.1864

____

j. 1.1075

____

h. 1.7774

____

THIRD REQUIREMENT: Evaluate the following expressions using logarithms.

k. 44 X 89

____

n. 0.011 X 0.54

____

l. 512 X 600

____

o. 0.00213 X 18

____

m. 100,000 X 1000

____

FOURTH REQUIREMENT: Evaluate the following expressions using logarithms.

p. 2259 ÷ 8

____

s. 1620 ÷ 67

____

q. 384 ÷ 8

____

t. 2 ÷ 83

____

r. 0.00854 ÷ 0.127

____

FIFTH REQUIREMENT: Find the square roots using logarithms.

u. 0.0006250

____

v. 2,250,000

____

w. 169

y. 121

x. 0.0001

Section V. SOLUTIONS TO EXERCISES

1-36. SOLUTIONS TO EXERCISES, SECTION I (PARAGRAPH 1-8)

a. Associative property (para 1-2b)

b. Commutative property (para 1-2a)

c. Inverse property for multiplication (para 1-2d)

d. Identity property for multiplication (para 1-2c)

e. Distributive property (para 1-2e)

f. Inverse property (para 1-2d)

g. Identity property for addition (para 1-2c)

h. Commutative property (para 1-2a)

i. Distributive property (para 1-2e)

j. Inverse property for addition (para 1-2d)

k. 18 (para 1-5a)

l. -15 (para 1-3a)

m. 16 (para 1-7)

n. 12 (para 1-7)

o. -1.27 or -19/15 (para 1-7)

p. 180 y + 24 (para 1-7)

q. 9 (para 1-7)

r. 0 (para 1-7)

s. 0.778 or 7/9 (para 1-7)

t. -2/8 or -1/4 (para 1-7)

1-37. SOLUTIONS TO EXERCISES, SECTION II (PARAGRAPH 1-14)

The reference for these exercises is paragraphs 1-9 through 1-14:

a. $c = 6$

b. $b = -4$

c. $r = 17$

d. $z = -2.4$

e. $k = -3.6$ or $-18/5$

f. $v = -5$

g. $r = 4$

h. $-r = 16$ or $r = -16$

i. $Y = 0$

j. $Y = \frac{12 - 4x}{6}$

k. $k = 0$

l. $p = -9$

m. $x = -7$

n. $x = -16$

o. $m = -15$

p. $k = 3$

q. $z = \frac{pqr}{ab}$

r. $c = \frac{rst + ghi}{ab}$

s. $x = \frac{defrst}{abcyz}$

t. $b = -c$

1-38. SOLUTIONS TO EXERCISES, SECTION III (PARAGRAPH 1-22)

a. 2.406×10^{-3} (para 1-18)

b. 4.742×10^{3} (para 1-18)

c. 1.463×10^{0} (para 1-18)

d. 7.77482×10^{2} (para 1-18)

e. 9.1×10^{1} (para 1-18)

f. 2.534×10^{-1} (para 1-18)

g. 119 (para 1-17a)

h. 128 (para 1-17b)

i. 36 (para 1-16)

j. 1,000 (para 1-17d)

k. 27 (para 1-17c)

l. 4,096 (para 1-17c)

m. 2.0×10^{-8} (para 1-21)

n. 4.0×10^{9} (para 1-21)

o. 7.48×10^{7} (para 1-20)

p. 6.804×10^{0} (para 1-20)

q. $\frac{7^4}{1}$ (para 1-19)

r. $\frac{1}{10^{16}}$ (para 1-19)

s. $\frac{5.5 \times 10^{-6}}{9.1 \times 10^{3}}$ (para 1-19)

t. $\frac{2^{-4}}{10^{8}}$ (para 1-19)

1-39. SOLUTIONS TO EXERCISES, SECTION IV (PARAGRAPH 1-35)

a. 5.8420 (paras 1-26 thru 1-28)

b. 2.9143 (paras 1-26 thru 1-28)

c. by calculator: -3.2007 (paras 1-26 thru 1-28)

by tables: $\bar{4}.7993$

d. 3.5340 (paras 1-26 thru 1-28)

e. by calculator: -1.3391 (paras 1-26 thru 1-28)

by tables: $\bar{2}.6609$

f. 0.0500 (para 1-30)

g. by calculator: 6.51×10^{-3}

by tables: $-2.1864 = \bar{3}.8136$

antilog $3.8136 = 6.51 \times 10^{-3}$ (para 1-30)

h. 59.9 (para 1-30)

i. 34.7 (para 1-30)

j. 12.8 (para 1-30)

k. 3.9×10^{3} (para 1-31)

l. 3.07×10^{5} (para 1-31)

m. 100,000,000 or 10^{8} (para 1-31)

n. 0.0059 (para 1-31)

o. 0.038 (para 1-31)

p. 282 (para 1-32)

q. 48 (para 1-32)

r. 0.067 (para 1-32)

s. 24.2 (para 1-32)

t. 0.024 (para 1-32)

u. 0.025 (para 1-33)

v. 1500 (para 1-33)

w. 13 (para 1-33)

x. 0.01 (para 1-33)

y. 11 (para 1-33)